Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		2113517

The Model of Decomposition of Wood Fiber by Fungi

Abstract

As a member of the decomposition process, fungi play an important role in the carbon cycle. To investigate the decomposition of wood fiber by fungi, scientists have carried out a lot of experiments, proposed a variety of models and made some achievements. Based on partially published data, this paper constructs a series of models to explore fungal inter specific interactions and fungal decomposition.

Model I: The Decomposition of Wood Fiber. This is the backbone of this paper. We believe that in a given environment, the decomposition capacity of the system is related to the number and the decomposition rate of various fungi. Based on this, we establish a simple and powerful differential equation to describe the decomposition of ground litter by fungi. And on this basis, we study the characteristics, interactions and quantity of fungi.

Model II: Trait of Fungi. Based on some available data, we found that there is a strong relationship between the decomposition rate of wood fiber by various groups of fungi and their growth rates (usually described as hyphal extension rate) and moisture tolerance. Using the idea of multiple regression, we established the relationship between the decomposition rate of fungi, the hyphal extension rate and the moisture tolerance, and roughly described the change of the decomposition rate of different types of fungi. Next, in order to describe different population combinations, we use the Monte Carlo method to randomly generate populations or combinations of populations in order to study it.

Model III: Fungal Interaction and Population. This is the main part of this paper. In this study, we first simulate the competition among populations by using a two-dimensional random cellular automata model, and studied the interactions of 3, 4 and 5 fungi. At the same time, we also establish a multivariate Gause-Lotka-Volterra model to describe the change of the equivalent number of multiple fungal populations. After many times of simulation, the quantity change relationship of different population combinations is obtained.

Model IV: Environmental Impact. We consider the influence of environmental factors (mainly temperature and humidity). Based on the existing data, the "environmental impact factor" is evaluated through the grey correlation analysis method, and the revised decomposition model is obtained. With the help of this model, we also obtain the maximum growth rate and decomposition rate of the fungal population under different climatic conditions.

Finally, we established the final model of fungal decomposition of wood fiber. On the one hand, we adjusted the number of fungi through the quantitative model. And on the other hand, we adjusted the growth rate and decomposition rate of fungi through the environmental model. We can analyze the impact of environmental change on the system and the significance of biodiversity. At last, we conducted stability analysis on our model the results are reliable.

Keywords: Fungi; Random Celluar Automata; Computer Simulation

Contents

1 Introduction

1.1 Problem Background

The carbon cycle plays an important role for the creatures on earth. And one of the crucial part of the carbon cycle is the decomposition of plant material and woody fibers, whose promoters are usually decomposers in ecosystems. The decomposition of wood by fungi is an important research object in this paper.

Figure 1: Scientist holding a dish of microbial

Recently, some researchers studied 34 kinds of different fungi from Armillaria gallica to Xylobolus subpileatus, and have discovered that some characteristics of fungi determine the rate at which they decompose wood. Meanwhile the relationship between these characteristics has been pointed out. The researchers also found that strains of some fungi tend to grow better under changes in humidity and temperature. Among them, the most remarkable characteristic of all kinds of fungi is growth rate (which is usually described by hyphal extension rate) and moisture tolerance.

What we are interested in is how fungi decompose wood in the ecosystem. At the same time, we also want to know how different fungal populations interact with each other. Finally, we also want to know how fungi compete and decompose wood in different environments and conditions. Therefore, we set up several different mathematical models to describe and predict this process.

1.2 Restatement of the Problem

- Set up the decomposition model of woody fibers in the presence of various fungi;
- In the above model, consider the interaction between different kinds of fungi. Before this, we need to study the characteristics of single fungus (including growth rate and moisture tolerance) and include them in the previous model;
- Analysis the established model, and describe the interaction between different fungi. In order to study the dynamic characteristics of the interaction, it is necessary to describe its short-term and

long-term trends. At the same time, our analysis should include the impact of rapid fluctuations in the environment, and combined with the trend of atmospheric change to assess the impact of changes in the local weather patterns;

- Consider the relative advantages and disadvantages of each species and different species combinations in the model, as well as different environments, including arid, semi-arid, temperate, arboreal and tropical rainforest, in order to analyze the situation of different environments;
- Describe how the diversity of fungal species affects the efficiency of a system in decomposing wood according to the model. The significance of biodiversity is explained when considering the changes of local environment.

1.3 Our Approach

In order to solve the problem, our main work is as follows.

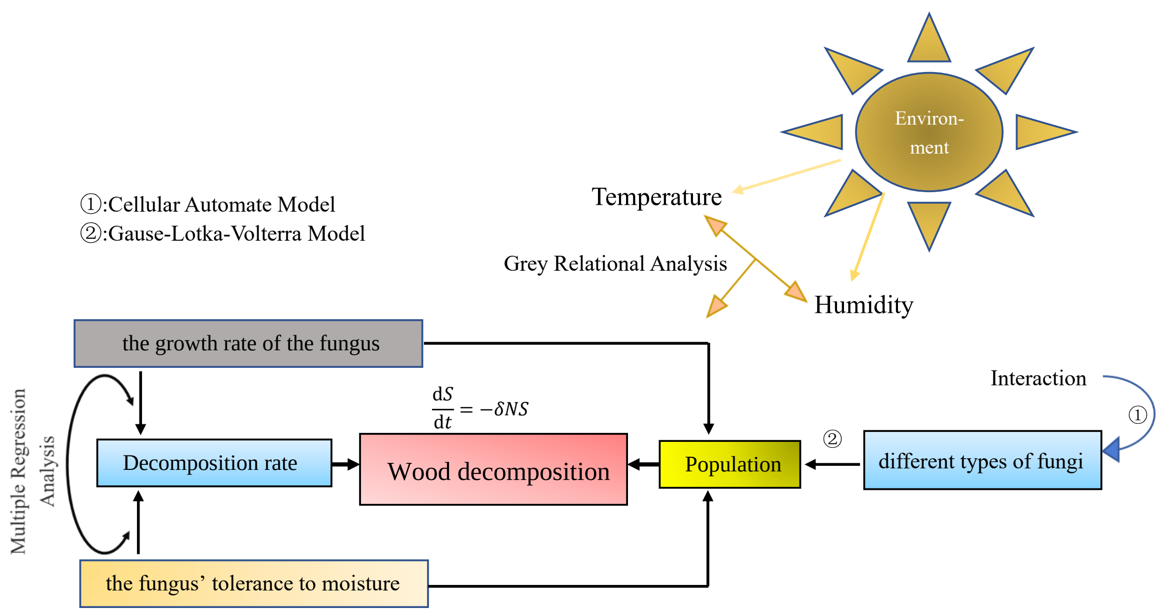


Figure 2: The model of fungal decomposition of wood fiber

- Set up the decomposition model of wood fiber;
- Found the relationship between growth rate, moisture tolerance and decomposition rate based on the trait data of various fungi;
- Set up the interaction models between different fungi;
- Set up the numerical model of fungal population combined with the previous interaction model;

- Analysis the influence of environmental change on fungi combined with the influence of different types of weather and climate;
- Analyzed and explained the significance of biodiversity combined with our model.

2 General Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:** The total decomposition rate of wood is related to the equivalent number and the decomposition rate of different microorganisms and is the weighted average of them;

  Reasons: Different species of fungi have their corresponding decomposition rate of wood, which is an important characteristic of fungi themselves. In order to describe the decomposition ability of all kinds of fungi, we assume that there is an equivalent number of different kinds of fungi.

- **Assumption 2:** A single kind of fungus can be identified by using its growth rate and moisture resistance;

  Reasons: Under certain conditions, different fungi correspond to a set of specific growth rates and moisture tolerance, which are the main factors affecting the decomposition rate of fungi. This paper mainly explores the decomposition of wood by fungi as a decomposing agent. Without loss of rationality, we believe that fungi are uniquely determined by its growth rate and moisture tolerance.

- **Assumption 3:** Different species of fungi use the same proportion of energy derived from the environment;

  Reasons: This is a commonly used assumption in ecology, so that each population has a certain limit (known as environmental capacity) in the case of limited environmental resources. From this, a Gause-Lotka-Volterra model can be established to predict population changes.

- **Assumption 4:** Changes in the environment are mainly reflected in changes in temperature and humidity.

  Reasons: The two most important parameters in the environment are temperature and humidity. Temperature indicates the intensity of molecular thermal movement in the environment, and also affects fungi directly or indirectly by affecting enzyme activity and so. Humidity represents the amount of water vapor in the environment, and different fungi have different water tolerances, which makes it important to consider humidity.

3 Model Preparation

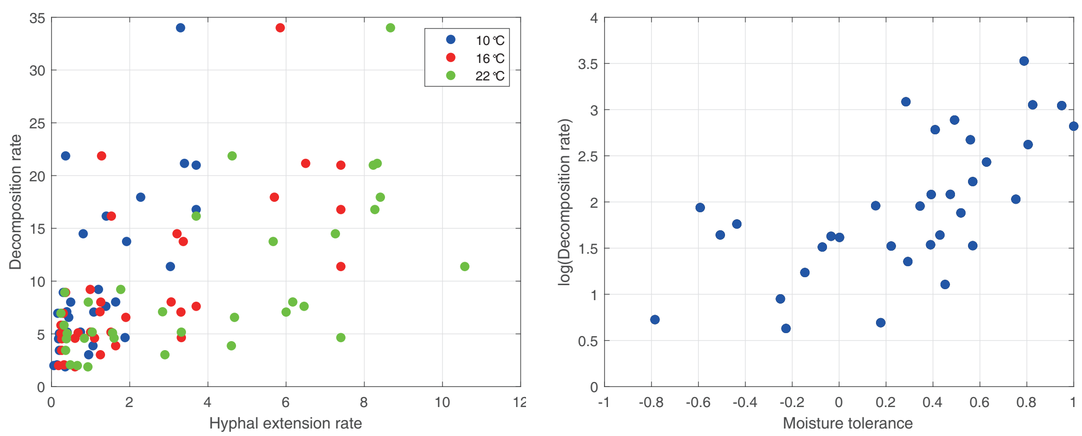
3.1 Notations

Table 1: Notations

Symbol	Description	Unit
S	Total amount of wood in the environment.	g
δ	Wood decomposition rate.	$\%$
α	Hyphal extension rate, which is used to describe growth rate.	$\mathrm{mm} \cdot \mathrm{day}^{-1}$
β	Moisture tolerance.	$\%$
N	Equivalent quantity.	-
T	Temperature	$^{\circ}\mathrm{C}$
M	Humidity	$\%$

3.2 Data Preprocessing

Here, set wood decomposition rate as δ, hyphal extension rate as α and moisture tolerance as β.



(a) The relationship between α and δ (b) The relationship between β and $\ln \delta$

Figure 3: The relationship between the traits of fungi

According to the data attached in Reference [1], we can get the wood decomposition rate and hyphal extension rate of 34 different isolates. Therefore, the relationship between hyphal extension rate and wood decomposition rate under different temperatures is obtained, as shown in the Figure 3(a).

Then according to the attached data in Reference [2], the moisture tolerance of 34 isolates can be obtained. Thus, the relationship between decomposition rate and moisture tolerance can be obtained and shown in the Figure 3(b).

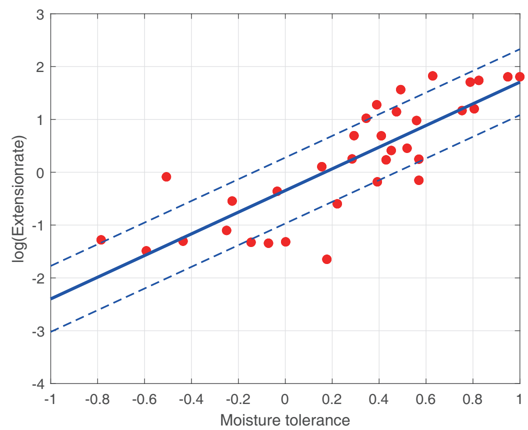


Figure 4: The relationship between $\ln\alpha$ and β

According to the above data, 34 different fungi have their own traits. Meanwhile, according to Reference [1], it can be found that $\ln\delta$ and $\ln\alpha$, $\ln\delta$ and β have certain linear relationship. At the same time, because of the above linear relationship, we can also get that $\ln\alpha$ and β have a linear relationship, as shown in the Figure 4.

The relationship between $\ln\alpha$ and β is obtained by

$$\ln\alpha = 2.0517\beta - 0.3459 \pm 0.6239.$$

Therefore, it is considered that growth rate α and moisture tolerance β are also related. In this paper, growth rate α and moisture tolerance β satisfy the above relationship for any possible fungal population.

4 Model Building

4.1 Decomposition of Wood Fiber

Our first and the final goal is to model the decomposition of wood fiber.

Let's set the total decomposition rate of wood as δ, which represents the proportion of dry weight of wood mass loss in 122 days, so that the wood mass loss in each amount of time of $\mathrm{d}t$ that each fungus contributes is $\delta S\mathrm{d}t$. Then, if the total amount of wood fiber in the soil is S and the numbers of fungi is N, the differential equation can be listed as

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\delta N S. \tag{1}$$

Equation (1) describes the variation rule of the total amount of wood fiber S with time t. If δ and N is considered to be a constant with time t, then start from the time of 0 and solve the differential equation (1), we have

$$S(t) = S_0 \exp(-\delta N t),$$

Where S_0 represents the initial weight of wood fiber.

In the soil, there are many kinds of fungi. We assume that the total decomposition rate of wood is related to the equivalent number of different fungi. As time changes, the equivalent number of different microbial also changes. So δ is not a constant number over time t. And it is clear that N is also changing over time.

Suppose there are n kinds of species of fungi. For $1 \le i \le n$, the equivalent number of each kind of fungi is $N_i(t)$, and the corresponding decomposition rate of wood is δ_i, then the total decomposition rate is

$$\delta(t) = \sum_{i=1}^{n} N_i(t)\delta_i.$$

Solve the differential equation (1), we have

$$S(t) = S_0 \exp\left(-\int_0^t \delta(s)\mathrm{d}s\right) = S_0 \exp\left(-\int_0^t \sum_{i=1}^{n} N_i(s)\delta_i \mathrm{d}s\right). \tag{2}$$

Equation (2) is a preliminary mathematical model, which we used to describe the breakdown of wood fibers under the action of different fungi.

4.2 Trait of Fungi

In Reference [1], we already know that $\ln\delta$ and $\ln\alpha$, along with $\ln\delta$ and β have some linear relationships. And in the process of modeling, in order to better explore the relationship between decomposition rate, growth rate and moisture tolerance, we further assume that $\ln\delta$ should meet a linear relationship with $\ln\alpha$ and β, i.e

$$\ln\delta = p\ln\alpha + q\beta + r + \varepsilon.$$

In the equation, p, q and r are constants unrelated to δ, α and β. And ε is an error, which changes with the value of δ, α and β. For the data set of independent observations $(\delta_i, \alpha_i, \beta_i)(1 \le i \le n)$, let

$$\boldsymbol{A} = \begin{pmatrix} \ln\alpha_1 & \beta_1 & 1 \\ \ln\alpha_2 & \beta_2 & 1 \\ \vdots & \vdots & \vdots \\ \ln\alpha_n & \beta_n & 1 \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \ln\delta_1 \\ \ln\delta_2 \\ \vdots \\ \ln\delta_n \end{pmatrix}.$$

To make the model as close as possible to the true value, using the idea of least square method, the error

$$e = \sum_{i=1}^{n} \varepsilon^2 = \sum_{i=1}^{n} (\ln\delta_i - p\ln\alpha_i - q\beta_i - r)^2$$

should be kept as small as possible. Let

$$\begin{cases} \dfrac{\partial e}{\partial p} = -2\sum_{i=1}^{n}(\ln\delta_i - p\ln\alpha_i - q\beta_i - r)\ln\alpha_i = 0 \\ \dfrac{\partial e}{\partial q} = -2\sum_{i=1}^{n}(\ln\delta_i - p\ln\alpha_i - q\beta_i - r)\beta_i = 0 \\ \dfrac{\partial e}{\partial r} = -2\sum_{i=1}^{n}(\ln\delta_i - p\ln\alpha_i - q\beta_i - r) = 0 \end{cases},$$

and thus we have

$$\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{\beta} = \boldsymbol{A}^T\boldsymbol{\gamma}.$$

Finally, we get the coefficients of the model

$$\hat{\boldsymbol{\beta}} = \begin{pmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{pmatrix} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{\gamma} = \begin{pmatrix} 0.3604 \\ 0.2468 \\ 1.8332 \end{pmatrix}.$$

Hence the growth rate, moisture tolerance and decomposition rate satisfy the Equation (3)

$$\ln\delta = 0.3604\ln\alpha + 0.2468\beta + 1.8332. \tag{3}$$

Based on the Equation (3) above, we have

$$\delta = 6.2538\cdot\alpha^{0.3604}\cdot\mathrm{e}^{0.2468\beta}.$$

Thus we have a description of the characteristics of all the classes of fungi, as Figure 5 shows.

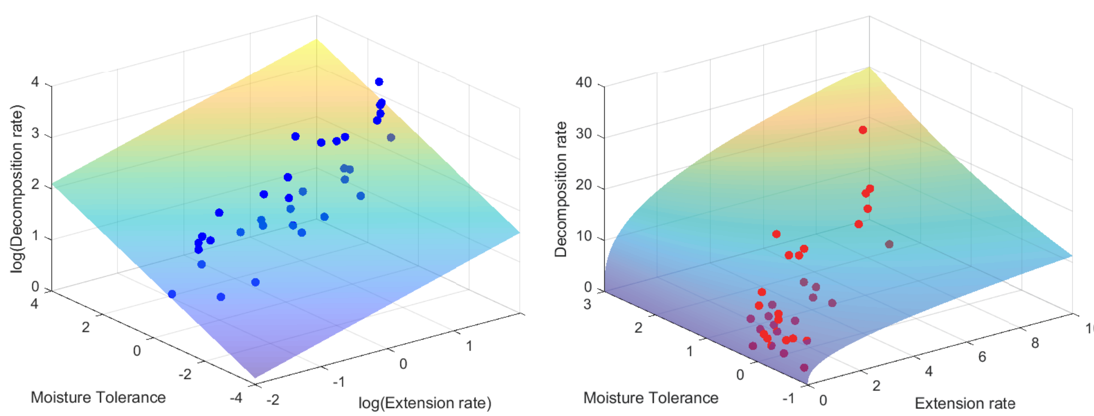


Figure 5: The relationship between the traits of fungi

4.3 Random Celluar Automata

Next, we use the computer to simulate, and try to use the model similar to the standard cellular automata to simulate the evolution of population size visually based on time series.

```
input: Amount,natural growth rate and adaptive index of species
output: Time-dependent curve of group sizes;Quantity graph
Allocates memory space for variables and initializes them
if Start button is pressed then
    Calculate the individual increment of each population under ideal conditions
    Generat new individuals of each population and the intraspecific competition
    for Every point in space do
        Calculate whether there are more than two species of individuals at the point
        if There are more than two species then
            Generate adaptive parameters for each population at this point
            Find the corresponding population with the largest adaptive parameter
            Other populations are judged to have failed in the competition,only the newly
              screened populations exist at this point
        end
    end
end
Calculate the size of all the populations in the graph
Reflect the existence of various groups at each point onto the graph
return Quantity graph and real-time data of various groups
Stores real-time data from each loop
if Pause key is pressed then
    Reset the pause and run keys
    Pause the operation of the loop
    return Time-dependent curve of various group sizes
end
```

The basic principle of this computer simulation is similar to that of standard cellular automata, but the core of its control rules is based on probability control of uniformly distributed random numbers, rather than an iterative structure controlled by a single cell and its surrounding environment.

The core of this algorithm includes three parts: reproduction, intraspecific competition and interspecific competition. Reproduction and intraspecific competition are controlled by the same code, and the probability control and growth model are consistent with the traditional Logistic growth model. Interspecific competition was controlled by a random parameter set, and the dominant species at the same locus were determined by comparing the adaptability indices of different species at the same locus. In the program, this index is represented as the product of the adaptive parameter, the existence parameter and the random parameter. We point out that the random parameters in this code are controlled by both the random factors and the properties of the species themselves, namely the adaptive parameters and the self-growth parameters. The theoretical structure and experimental results of this model are consistent with the general cognitive law.

This model still has some limitations. First of all, it can not give the time-dependent variation law of the spatial distribution of the population, but only give the approximate succession picture of the population size. Secondly, this model is established on the premise that the population's heredity is stable and there is basically no large-scale heredity and variation. More sophisticated algorithms, such as standard cellular automata modified by genetic algorithms, may be needed to provide a more detailed description of the spatially varying picture of population.

4.4 Gause-Lotka-Volterra Model

In the following process of model establishment, we believe that the interaction between populations and the change of environment will directly affect the equivalent number of fungi N_i, thus affecting the decomposition rate of wood fiber δ.

In order to describe the interaction relationship between different populations and reflect the competition among populations, we established the extended Gause-Lotka-Volterra model, namely the differential equations

$$\begin{cases} \dfrac{\mathrm{d}N_1}{\mathrm{d}t} = \dfrac{\alpha_1 N_1}{K_1}(K_1 - \sigma_{11}N_1 - \sigma_{12}N_2 - \cdots - \sigma_{1n}N_n) \\ \dfrac{\mathrm{d}N_2}{\mathrm{d}t} = \dfrac{\alpha_2 N_2}{K_2}(K_2 - \sigma_{21}N_1 - \sigma_{22}N_2 - \cdots - \sigma_{2n}N_n) \\ \quad\vdots \\ \dfrac{\mathrm{d}N_n}{\mathrm{d}t} = \dfrac{\alpha_n N_n}{K_n}(K_n - \sigma_{n1}N_1 - \sigma_{n2}N_2 - \cdots - \sigma_{nn}N_n) \end{cases},$$

where K_i represents the environmental capacity when there is only one species. To obtain the environmental capacity of a single population, we consider that the energy obtained by a population from the environment is all used for the growth of the population, so as to approximately consider $K_i \approx \dfrac{\alpha_i S}{\delta_i}$. In order to simplify the problem and calculate in MATLAB, set

$$\boldsymbol{D} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}, \boldsymbol{N} = \begin{pmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{pmatrix}, \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{pmatrix},$$

where $\sigma_{ij} = \dfrac{\delta_j}{\delta_i}$ represents the intensity of competition between two species. We have

$$\begin{cases} \dfrac{\mathrm{d}N_1}{\mathrm{d}t} = \dfrac{N_1}{S}(\alpha_1 S - \delta_1 N_1 - \delta_2 N_2 - \cdots - \delta_n N_n) \\ \dfrac{\mathrm{d}N_2}{\mathrm{d}t} = \dfrac{N_2}{S}(\alpha_2 S - \delta_1 N_1 - \delta_2 N_2 - \cdots - \delta_n N_n) \\ \quad\vdots \\ \dfrac{\mathrm{d}N_n}{\mathrm{d}t} = \dfrac{N_n}{S}(\alpha_n S - \delta_1 N_1 - \delta_2 N_2 - \cdots - \delta_n N_n) \end{cases}. \tag{4}$$

Give a set of initial values, and we can simulate the quantity change of multiple populations by equation (4). Last, considering the overall decomposition rate of these fungal components

$$\delta = \sum_{i=1}^{n} N_i(t)\delta_i.$$

Thus, the decomposition rate of different systems can be estimated.

4.5 Grey Relational Analysis of Environmental Impact

We assume that the change of environmental humidity mainly affects the system by changing the growth rate of a certain population. Suppose the ambient temperature is T, humidity is M, and the optimal temperature and the humidity is T_0 and M_0. Let some index of the environment is D, and the change of D is ΔD.

According to the definition, β_i is a number in $[-1, 1]$. It is also noted that if β_i is larger, the species is more tolerant to environmental change. Therefore, let $\beta_i' = \dfrac{\beta_i + 1}{2}$. Then the larger β_i' is, the stronger tolerance of the species to environmental change is. If α is reduced to half of its original value, the proportion of the amount changed is

$$\frac{\Delta D}{D} \approx \frac{\beta_i'}{2} = \frac{\beta_i + 1}{4}.$$

On the basis of that, we have

$$\ln \alpha_i' - \ln \alpha_i = \ln \frac{1}{2} = k\frac{\Delta D}{D},$$

hence $k = \dfrac{-4\ln 2}{\beta_i + 1}$. So there is a variation relationship between growth rates and environmental change

$$\ln \alpha_i' = \ln \alpha_i - \frac{4\ln 2}{\beta_i + 1} \cdot \frac{|\Delta D|}{D}.$$

To explore the effects of the environment on fungal decomposition, there are some relevant data. During the data processing, we were surprised finding that changes in hyphal extension rate α was associated with humidity M and temperature T. In order to quantify the strength of the impact of both on hyphal extension rate, we processed the relevant data using a gray relational analysis model. We analyzes the correlation between different indicators and hyphal extension rate.

Taking the hyphal extension rate as the reference series, the results are obtained as

$$\{a(i) : 1 \le i \le n\}.$$

Taking the indicators (temperature & humidity) of environment as comparison series, we get

$$\{x_m(i) : 1 \le i \le n\}, m = 1, 2,$$

where we substitute $\{x_1(i)\}$ for temperature and substitute $\{x_2(i)\}$ for humidity.

Considering the different dimensions of the two indicators. Before comparison, they should be initialized to

$$\{y_m(i), 1 \le i \le n\} = \left\{1, \frac{x_m(2)}{x_m(1)}, \frac{x_m(3)}{x_m(1)}, \cdots, \frac{x_m(n)}{x_m(1)}\right\}.$$

Here for $1 \le i \le n$ and for a specific species, the correlation coefficient of the comparison sequence to the reference sequence is obtained as

$$\xi_{k,m}(t) = \frac{\min_k \min_i |a_k(i) - y_m(i)| + \rho \max_k \max_i |a_k(i) - y_m(i)|}{|a_k(t) - y_m(t)| + \rho \max_k \max_i |a_k(i) - y_m(i)|}.$$

In the formula, ρ represents the correlation coefficient and put $\rho = 0.5$ here.

Next, we can calculate the indicator

$$r_{k,m}^{(0)} = \frac{1}{n} \sum_{t=1}^{n} \xi_{k,m}(t).$$

In order to illustrate the impact of each indicator more reasonably, normalization is required. That is

$$r_{k,m} = r_{k,m}^{(0)} / \sum_{m=1}^{2} r_{k,m}^{(0)}.$$

After calculating the results, we have

$$\frac{\Delta D}{D} = 0.5138 \cdot \frac{|T - T_0|}{T} + 0.4862 \cdot \frac{|M - M_0|}{M}.$$

Combining the above conclusions, we get the correlation

$$\ln \alpha' = \ln \alpha - \frac{4 \ln 2}{\beta + 1} \cdot \left(0.5138 \cdot \frac{|T - T_0|}{T} + 0.4862 \cdot \frac{|M - M_0|}{M}\right). \tag{5}$$

Thus, by grey prediction model, we have successfully established the influence model of environmental temperature T and humidity M on the growth rate α.

4.6 The Final Decomposition Model

In order to obtain the final decomposition model of wood fiber, we considered the organic integration of the above model. Let the ideal temperature of fungus be T_0, ideal humidity be M_0, and there is a piece of soil whose temperature and humidity are T and M. The initial total quantity of wood fiber in the soil is S_0. After a period of time t, the quantity of wood at any moment is $S(t)$. There is a total population of n species of fungi in the environment, where the growth rate is α_i, the moisture tolerance is β_i, and the decomposition rate of wood is δ_i. The equation can be listed as

$$\begin{cases} S(t) = S_0 \exp\left(-\int_0^t \sum_{k=1}^{n} N_k(s)\delta_k \mathrm{d}s\right) \\ \dfrac{\mathrm{d}N_i}{\mathrm{d}t} = \dfrac{N_i}{S_0}\left(\alpha_i' S_0 - \sum_{k=1}^{n} \delta_k N_k\right) \\ \ln \alpha_i' = \ln \alpha_i - \dfrac{4\ln 2}{\beta_i + 1} \cdot \left(0.5138 \cdot \dfrac{|T - T_0|}{T} + 0.4862 \cdot \dfrac{|M - M_0|}{M}\right) \\ \ln \delta_i = 0.3604 \ln \alpha_i' + 0.2468\beta_i + 1.8332 \end{cases}. \tag{6}$$

In addition, if we need to study the interaction of fungi on the plane, we can use the two-dimensional random cellular automata model established by us, and use the computer to simulate.

This is **the final model of fungal decomposition of wood fiber** that we established, according to the requirements of the topic.

5 Results of the Model

5.1 Geometric Prediction of Interactions

Several mathematical simulations have been carried out for interspecific competition patterns in a limited space with only 3, 4 and 5 populations under different parameters. In the following, we presents three visualizations of population size and three visualizations of time-dependent evolution of population size.

We can see that these results are consistent with the general cognitive law and meet the our expectation. The corresponding parameters of these patterns are noted in Figure 6.

It is noted that this algorithm is still bionic and time-varying in nature, so the computational amount is much larger than that of the traditional differential equation model, and the computational efficiency is low. At the same time, the statistical fluctuation will also have an impact on this simulation. Therefore, when predicting short-term and long-term trends, the Gause-Lotka-Volterra model is mainly used for calculation, and this algorithm is used for pseudo-true verification.

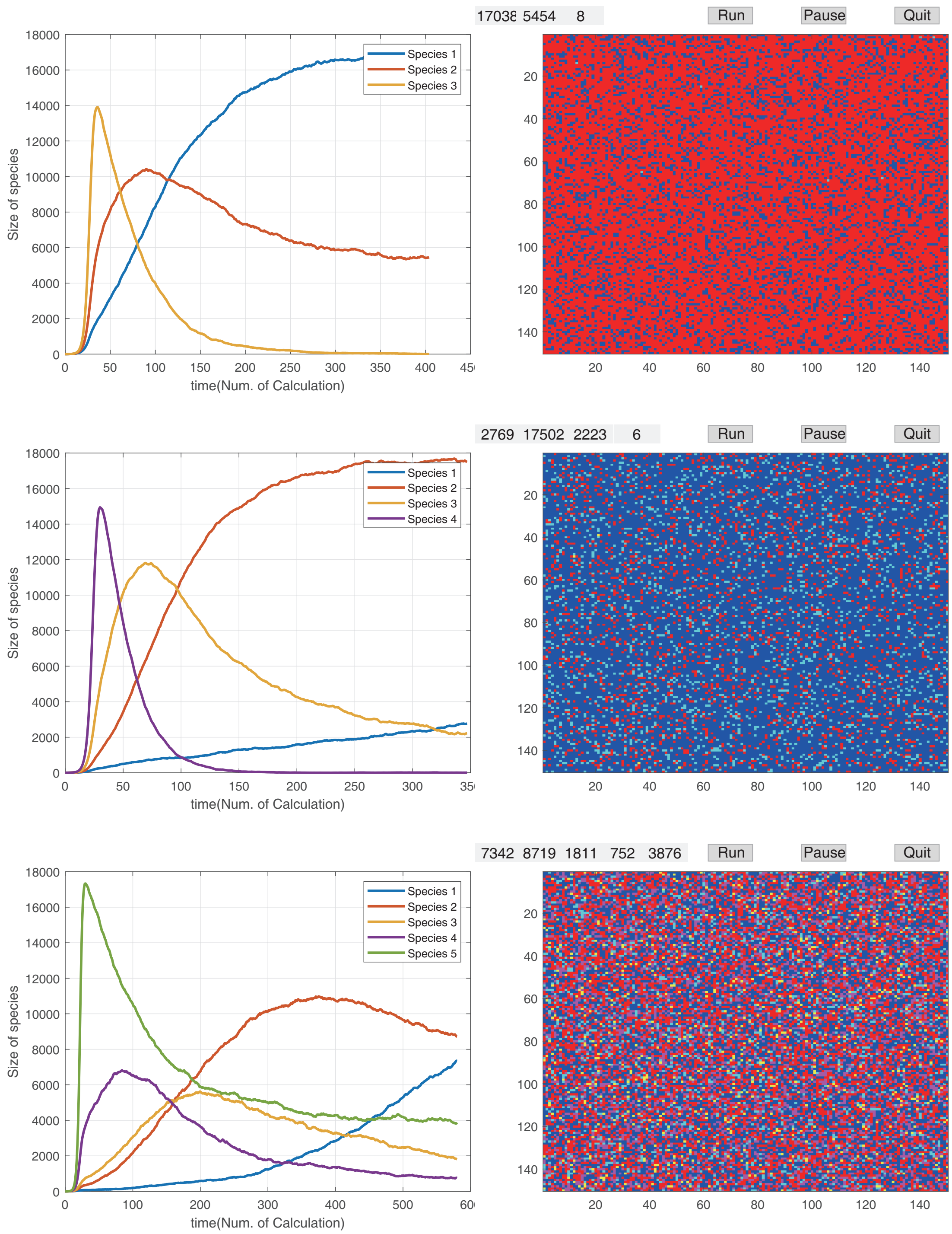
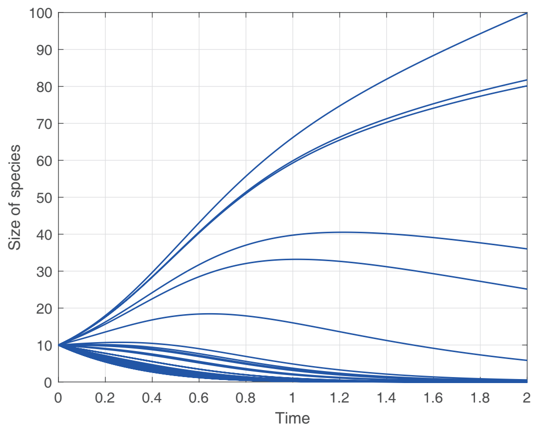


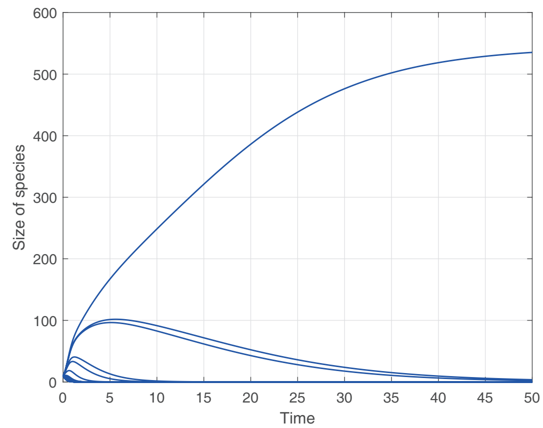
Figure 6: Results of random celluar automata. The parameter of species used in the simulation are: selfrate=[0.28,0.35,0.42]; condirate=[0.41;0.35;0.29]; selfrate=[0.2,0.3,0.4,0.5]; condirate=[0.6;0.5;0.4;0.3]; selfrate=[0.15,0.25,0.33,0.45,0.58]; condirate=[0.967;0.7;0.58;0.478;0.428];

5.2 Short Term and Long Term Trends

For the 34 species of fungi given in the Reference [1], the results obtained under ideal temperature and humidity are shown in Figure 7. The figure 7(a) below shows the results in the short term. It can be seen that some species have no competitive advantage in the short term, while some species have certain growth. Meanwhile, Figure 7(b) shows the long-term results, with only one competitive fungus growing over a long period of time, which fits our intuition.



(a) Short term trend



(b) Long term trend

Figure 7: Results of Gause-Lotka-Volterra model

The ultimate winner, according to the predictions, was a fungus called Phlebiopsis flavidoalba FP150451 A8G. And after many simulations here, we get that the total population with a higher growth rate α will have a greater final competitive advantage, which is in line with the ecological reality.

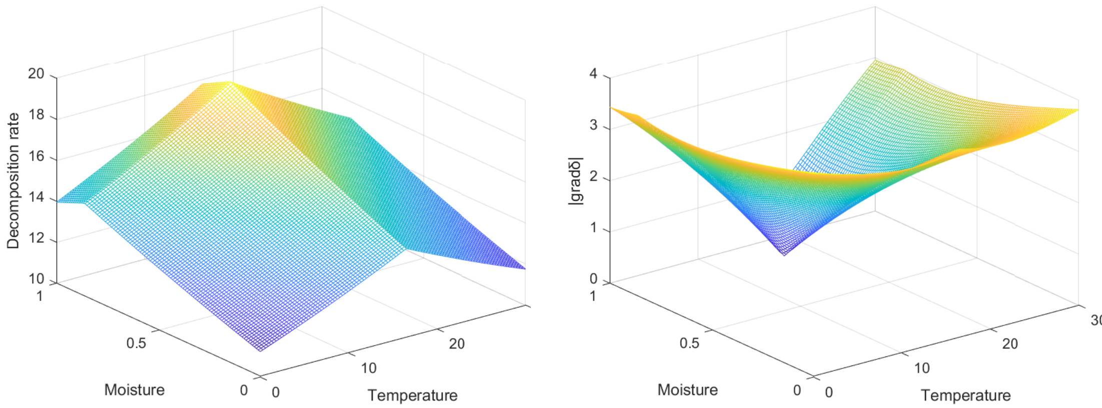
5.3 Situation in Different Environments

Based on the model we have established, we can plot the relationship between the maximum decomposition rate and temperature and humidity, as figure 8(a) shows.

Among them, there is a certain temperature and humidity, so that the decomposition rate reaches the maximum, which is consistent with reality. Next, to analyze the effects of environmental change, we calculate the modulus of the gradient

$$|\text{grad}\delta| = \sqrt{\left(\frac{\partial \delta}{\partial M}\right)^2 + \left(\frac{\partial \delta}{\partial T}\right)^2}$$

and take it as the impact of environmental changes. The calculated results are shown in figure 8(b). It can be seen from the image that the system is less sensitive to environmental changes near the point where the decomposition rate reaches the maximum. The further away from the maximum point, the more sensitive the system is to environmental changes.



(a) The decomposition rate (b) The gradient

Figure 8: Results of grey relational analysis model

Finally, according to the type of environment given in the question, we can infer its representative temperature and representative humidity.The calculated results are shown in table **??**.

Table 2: The situation in different environments

Type	Temperature	Humidity	Growth rate	Decomposition rate
arid	30	0%	4.6753	13.9551
semi-arid	30	20%	5.9738	15.2438
temperate	30	50%	8.6282	17.4035
temperate	20	50%	8.2202	17.1024
arboreal	30	70%	7.6330	16.6516
arboreal	20	70%	7.2721	16.3635
tropical rain forests	30	90%	5.9738	15.2438

In Table 2, we can see that fungus growth is higher in temperate, and the corresponding decomposition rate is higher, which is in line with reality.

5.4 The Significance of Biodiversity

First, let's analyze **the change of decomposition rate caused by the increase of species number**. In the population number prediction model, we try to change the value of species number n in order to reflect the influence of species number change on the system. Set the same initial value $N_i(0) = \frac{10}{n}$, and take the total amount of environmental resources $S = 1000$ to calculate the corresponding total decomposition rate when $t = 2$. Calculate 1000 times and take the average value of the calculated results. After processing enough data and plotting the total decomposition rate and n, we can get a monotonically increasing and downward convex curve, as figure 9 shows.

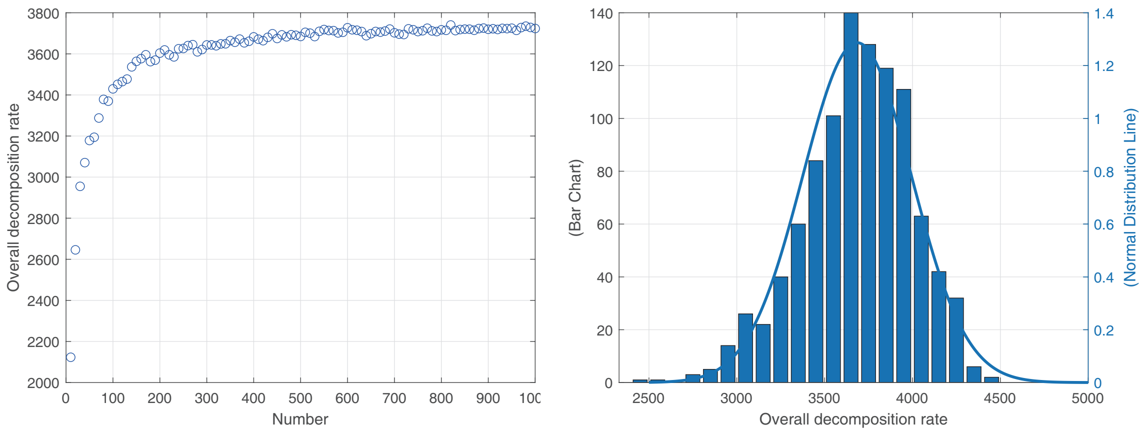


Figure 9: The change of decomposition rate caused by the increase of species number

The monotony indicates that, as the value of n keeps increasing, the total decomposition rate of the system keeps increasing; The concavity and convexity of the curve indicates that, with the increasing value of n, the growth range of the curve is also decreasing. There is a certain marginal diminishing effect, which is in line with the reality.

Second, we analyze **the ability of a system to resist risk as it becomes more diverse**. According to the above model, as the number of biological species n continues to grow, our system will become more complex, with more fungal species whose growth rates α and moisture tolerance β vary. Thus, there are some fungi, which have a higher growth rate and a certain competitive advantage, and thus increase in number. After sudden changes in the environment, even if the mortality rate increases, they will still retain a certain number.

To sum up, we can conclude that **biodiversity is of great significance to the system**.

6 Conclusion of the Model

6.1 Summary

To solve the first problem, we established the decomposition model of wood fiber, namely the differential equation

$$S(t) = S_0 \exp\left(-\int_0^t \sum_{i=1}^{n} N_i(s)\delta_i \mathrm{d}s\right).$$

In this model, how wood fibers decompose in the presence of multiple fungi is described.

To solve the second problem, we first obtained the characteristics of the individual fungi, and we find the linear relationship

$$\ln\delta = 0.3604\ln\alpha + 0.2468\beta + 1.8332.$$

Next, in order to describe the interaction, we simplified the standard cellular automata model and established a two-dimensional stochastic cellular automata model.

To solve the third problem, We first set up the quantity prediction model of the population, that is, the differential equations

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = \frac{N_i}{S}(\alpha_i S - \delta_1 N_1 - \delta_2 N_2 - \cdots - \delta_n N_n), 1 \leq i \leq n.$$

The number trend of many different populations can be predicted clearly and efficiently according to this model. And in order to quantitatively analyze the environmental impact, we establish a growth rate adjustment model

$$\ln\alpha' = \ln\alpha - \frac{4\ln 2}{\beta+1}\cdot\left(0.5138\frac{|T-T_0|}{T} + 0.4862\frac{|M-M_0|}{M}\right).$$

To solve the fourth problem, we associate it with Monte Carlo method and randomly select $n$groups of (α,β) to reflect different population combinations and analyze the corresponding situation. And we use our models in different climates.

To solve the fifth problem, we combined existing models. In order to illustrate the importance of biodiversity, we first analyze how biodiversity affects the efficiency of a system in decomposing wood. The second is to consider the importance of biodiversity when the local environment changes.

In conclusion, **we successfully builded the decomposition model of wood fiber, and used it to solve the practical problem.**

6.2 Stability Analysis

Consider the variety of fungi in the process of model establishment. We can simulate the combination of possible fungal populations by random number. Statistically, when the number of simulations is sufficient, the simulation results would be normal distribution. We consider **the stability of the simulation results**, and here let e be an error and satisfy

$$\delta = \bar{\delta} \mp e.$$

In figure 10, the blue line represents the average of the simulation results, and the red area represents the changing range of δ. As you can see, when the population is large enough, the results is going stable, which is consistent with our perception. **This indicates that our model can handles population interactions with reliable results.**

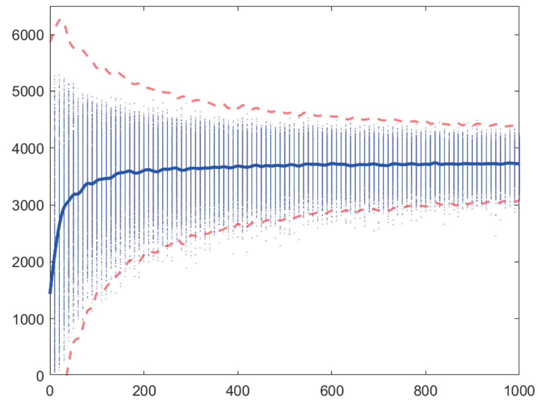


Figure 10: The simulation results

6.3 Strength

In the process of building the model, we went through the layers and built a fungal decomposition model according to our own ideas.

- We do not choose a fixed species to study, but use random number to simulate the combination of species under certain constraints, in view of the fact that there are many species of fungi;
- We consider the main influencing factors and reasonably simplify the model in practice. For example, when we counted the number of fungi, we chose the equivalent number instead of the absolute number, which effectively simplified the model;
- We conduct a large number of tests when solving the model, and found that our results have considerable stability. It can reflect the interaction and decomposition characteristics of fungi;

6.4 Possible Improvements

Some possible improvements are as follows.

- Collect more data through experiments, so as to give more reasonable parameters;
- Consider introducing temperature tolerance to better determine the environmental impact;
- Consider more complex factors for building a more elaborate model.

The Model of Decomposition of Wood Fiber by Fungi

Fungi often appear as the decomposer in the ecosystem, which plays a very important role in the energy flow and material circulation in nature, and people's production and living activities are inseparable from fungi. Therefore, it is very important to explore the natural properties of fungi, and solving this problem is helpful to improve the production level and quality of life of human beings. An important measure of the natural properties of fungi is the decomposition efficiency of fungi, that is, the total amount of organic matter that can be decomposed by fungi per unit time. In forest ecosystems, this index is also reflected in the decomposition efficiency of wood fiber. Naturally, the decomposing efficiency of fungi in an ecosystem is the sum of the decomposing efficiency of all fungi in that ecosystem.

Studies have shown that the lignocellulosic decomposition efficiency of fungi in a forest ecosystem is directly proportional to both the total number of fungi in the ecosystem and the total mass of lignocellulosic residues in the ecosystem. Unfortunately, the ratio of lignofiber decomposition efficiency to the product of the above two parameters is not a constant, but a parameter related to many parameters. The study of this functional relation can be divided into two subproblems. The first is to study the microcosmic problem of the relationship between the decomposition efficiency of fungi and the environment, the second is to study the macroscopic problem of the evolution law of the population size of various fungi in the ecosystem.

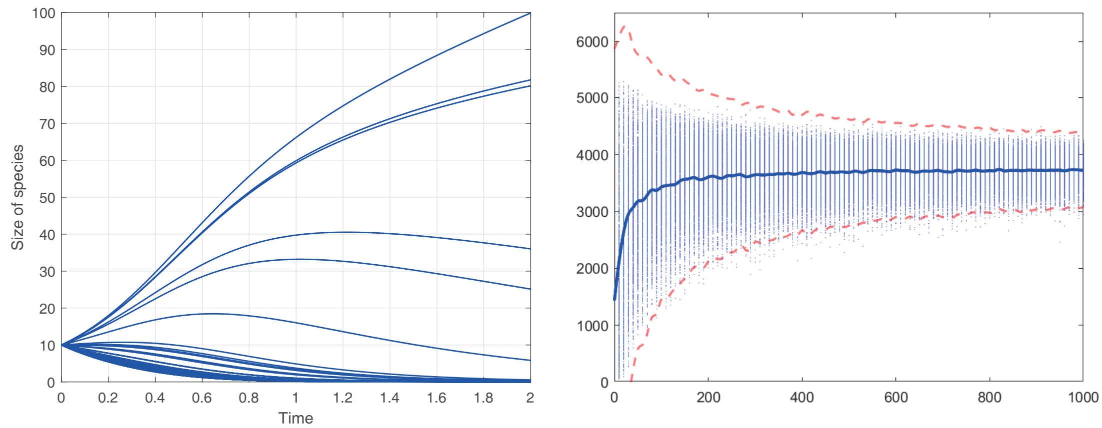
(1) The relationship between the decomposition efficiency of fungi and the parameters

With modern detection techniques and computational theories, we can now roughly predict the decomposition efficiency of a fungus based on several environmental parameters. The empirical study and data analysis showed that the logarithm of the maximum decomposition efficiency of fungi could be expressed linearly by the logarithm of its natural growth rate, water tolerance parameters and a specific constant. It can be seen that the effect of water tolerance parameters on the maximum decomposition efficiency of fungi is exponential, which is significantly greater than the effect of natural growth rate on the decomposition efficiency of fungi. This empirical formula is $\delta = 6.2538 \cdot \alpha^{0.3604} \cdot \mathrm{e}^{0.2468\beta}$.

We notice that fungi do not reach their maximum decomposition efficiency all the time, because the temperature and humidity of the environment will have a certain effect on the natural growth rate of fungi. In general, each fungus has a set of optimal temperatures and humidity. Under the optimal conditions determined by this set of parameters, the natural growth rate of fungi is maximized. According to statistical theory and measured data, we know that the actual optimal absolute difference of logarithm of fungal natural growth rate can be linearly expressed by the actual optimal absolute difference of temperature and humidity, and the relationship between them is like this $\ln \alpha' = \ln \alpha - \frac{4 \ln 2}{\beta + 1} \cdot \left(0.5138 \frac{|T - T_0|}{T} + 0.4862 \frac{|M - M_0|}{M} \right)$.

(2) Evolution of the population size of various fungi in the ecosystem

Let's start with the case of a single fungus. If there were only one species of fungus in the



ecosystem, then we would only need to consider the reproduction and intraspecific competition of the fungus – and as you might guess, this intraspecific competition behavior is well described by a Logistic growth retardation model that takes into account both factors. As we introduce more fungi into the ecosystem, things will get complicated and we will have to consider interspecies competition. At this point, I think you can easily understand the following two facts. First, the inhibition effect of interspecific competition on population size growth is positively correlated with the product of the total number of individuals in the two populations; second, the ecosystem does not allow all species to reach the environmental tolerance under ideal conditions. Based on these two facts, we can draw a differential equation describing the time-dependent evolution of population size:

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = \frac{N_i}{S}(\alpha_i S - \delta_1 N_1 - \delta_2 N_2 - \cdots - \delta_n N_n), 1 \leq i \leq n.$$

The numerical solution of this differential equation can effectively predict the evolution law of the population. Of course, the probabilistic control cellular automata model based on the above principle can also be used to predict its evolution law. Students who are interested in it can find relevant papers and conduct simulation. After running a series of simulations, we found that it is very difficult to find large groups of fungi that can coexist harmoniously in nature. There are obvious dominant species in the evolution of the fungal community, and these dominant species are replaced with time.

So far, we have found a suitable way to describe a fungus. Based on this principle, we can have some more in-depth discussion.

(3) The significance of biodiversity

In the simulated calculation, we found that under the condition of the significant increase in the number of fungal species, without considering the statistical fluctuation, the total decomposition efficiency of the ecosystem showed an upward trend, and the upward pattern was similar to the logarithmic curve. This seems to lead us to some conclusions – that biodiversity, for example, not only increases the stability of ecosystems, but also speeds up the circulation of matter and energy.

References

[1] Lustenhouwer N , Maynard D S , Bradford M A , et al. A trait-based understanding of wood decomposition by fungi[J]. Proceedings of the National Academy of Sciences, 2020, 117(21):201909166.

[2] Maynard D S , Bradford M A , Covey K R , et al. Consistent trade-offs in fungal trait expression across broad spatial scales[J]. Nature Microbiology, 2019.

A Appendix: Tools and Software

Paper written and generated via LaTeX, free distribution.
Graph generated and calculation using MATLAB R2019b.

B Appendix: The Codes

```
%Random_Celluar_Automata
clear;clc;
plotbutton=uicontrol('style','pushbutton','string','Run', 'fontsize',12, ...
    'position',[250,400,50,20], 'callback', 'run=1;');
erasebutton=uicontrol('style','pushbutton','string','Pause','fontsize',12,' ...
    position',[350,400,50,20],'callback','freeze=1;');
quitbutton=uicontrol('style','pushbutton','string','Quit','fontsize',12,' ...
    position',[450,400,50,20],'callback','stop=1;close;');
for i=1:5
    number(i) = uicontrol('style','text','string','0','fontsize',12, ...
        'position',[45*i-45,400,50,20]);
end
n=150;pic=ones(n,n,3)*255;img=image(pic);indi=zeros(n,n,5);
colour1=[255;0;0];colour2=[0;0;255];colour3=[0;128;128];colour4=[63;192;0]; ...
    colour5=[63;0;192];
colour=[colour1,colour2,colour3,colour4,colour5];control=1;sum=ones(5); ...
    num=zeros(5);
selfrate=[0.15,0.25,0.33,0.45,0.58];condirate=[0.967;0.7;0.58;0.478;0.428]; ...
    stop=0;run=0;freeze=0;
while stop==0
    if run==1
        for i=1:5
            num(i)=floor(selfrate(i)*sum(i))+1;
        end
        for i=1:5
```

```
        for j=1:num(i)
            indi(randi(n),randi(n),i)=1;
        end
    end
    for i=1:n
        for j=1:n
            tmp=0;
            for t=1:5
                tmp=tmp+indi(i,j,t);
            end
            if tmp>1
                randtmp=zeros(5);randrank=1:5;
                for t=1:5
                    randtmp(t)=indi(i,j,t).*condirate(t).*rand();
                end
                for m=2:5
                    if randtmp(1)<randtmp(m)
                        a=[randtmp(1),randrank(1)];
                        randtmp(1)=randtmp(m);randrank(1)=randrank(m);
                        randtmp(m)=a(1); randrank(m)=a(2);
                    end
                end
                for t=1:5
                    if randrank(1)==t
                        indi(i,j,t)=1;
                    else
                        indi(i,j,t)=0;
                    end
                end
            end
        end
    end
    sum=zeros(5);
    for t=1:5
        for i=1:n
            for j=1:n
                if indi(i,j,t)==1
                    sum(t)=sum(t)+1;
                end
            end
        end
    end
    for i=1:n
        for j=1:n
            for t=1:5
```

```
                    if indi(i,j,t)==1
                        for k=1:3
                            pic(i,j,k)=colour(k,t);
                        end
                        break;
                    end
                end
            end
        end
        figure(1)
        img=image(pic);
        control=control+1;controlvector(control)=control;
        for i=1:5
            sumtim(control,i)=sum(i);
            set(number(i),'string',num2str(sum(i)));
        end
    end
    if freeze==1
        run=0;freeze=0;
        for i=1:5
            figure(2)
            p(i)=plot(controlvector,sumtim(:,i),'linewidth',2);
            hold on
            axis on
            grid on
        end
        legend(p,{'Species 1','Species 2','Species 3','Species 4','Species ...
            5'});
        xlabel('time(Num. of Calculation)');ylabel('Size of species');
    end
end
```